Systems of Equations - Mixture Problems

Objective: Solve mixture problems by setting up a system of equations.

One application of systems of equations are mixture problems. Mixture problems are ones where two different solutions are mixed together resulting in a new final solution. We will use the following table to help us solve mixture problems:

	Amount	Part	Total
Item 1			
Item 2			
Final			

The first column is for the amount of each item we have. The second column is labeled "part". If we mix percentages we will put the rate (written as a decimal) in this column. If we mix prices we will put prices in this column. Then we can multiply the amount by the part to find the total. Then we can get an equation by adding the amount and/or total columns that will help us solve the problem and answer the questions.

These problems can have either one or two variables. We will start with one variable problems.

Example 1.

A chemist has 70 mL of a 50% methane solution. How much of a 80% solution must she add so the final solution is 60% methane?

	Amount	Part	Total
Start	70	0.5	
Add	x	0.8	
Final			

Set up the mixture table. We start with 70, but don't know how much we add, that is x. The part is the percentages, 0.5 for start, 0.8 for add.

	Amount	Part	Total
Start	70	0.5	
Add	x	0.8	
Final	$70+x$	0.6	

Add amount column to get final amount. The part for this amount is 0.6 because we want the final solution to be 60% methane.

	Amount	Part	Total
Start	70	0.5	35
Add	x	0.8	$0.8x$
Final	$70+x$	0.6	$42+0.6x$

Multiply amount by part to get total.
be sure to distribute on the last row: $(70+x)0.6$

$$35+0.8x=42+0.6x$$ The last column is our equation by adding

$$\underline{-0.6x \qquad -0.6x}$$ Move variables to one side, subtract $0.6x$

$$35+0.2x=42$$ Subtract 35 from both sides

$$\underline{-35 \qquad -35}$$

$$\frac{0.2x}{0.2}=\frac{7}{0.2}$$ Divide both sides by 0.2

$$x=35$$ We have our x!

35 mL must be added Our Solution

The same process can be used if the starting and final amount have a price attached to them, rather than a percentage.

Example 2.

A coffee mix is to be made that sells for \$2.50 by mixing two types of coffee. The cafe has 40 mL of coffee that costs \$3.00. How much of another coffee that costs \$1.50 should the cafe mix with the first?

	Amount	Part	Total
Start	40	3	
Add	x	1.5	
Final			

Set up mixture table. We know the starting amount and its cost, \$3. The added amount we do not know but we do know its cost is \$1.50.

	Amount	Part	Total
Start	40	3	
Add	x	1.5	
Final	$40+x$	2.5	

Add the amounts to get the final amount.
We want this final amount to sell for \$2.50.

	Amount	Part	Total
Start	40	3	120
Add	x	1.5	$1.5x$
Final	$40+x$	2.5	$100+2.5x$

Multiply amount by part to get the total.
Be sure to distribute on the last row $(40+x)2.5$

$$120+1.5x=100+2.5x$$ Adding down the total column gives our equation

$$\underline{-1.5x \qquad -1.5x}$$ Move variables to one side by subtracting $1.5x$

$$120=100+x$$ Subtract 100 from both sides

$$\underline{-100-100}$$

$$20=x$$ We have our x.

20mL must be added. Our Solution

World View Note: Brazil is the world's largest coffee producer, producing 2.59 million metric tons of coffee a year! That is over three times as much coffee as second place Vietnam!

The above problems illustrate how we can put the mixture table together and get an equation to solve. However, here we are interested in systems of equations, with two unknown values. The following example is one such problem.

Example 3.

A farmer has two types of milk, one that is 24% butterfat and another which is 18% butterfat. How much of each should he use to end up with 42 gallons of 20% butterfat?

	Amount	Part	Total
Milk 1	x	0.24	
Milk 2	y	0.18	
Final	42	0.2	

We don't know either start value, but we do know final is 42. Also fill in part column with percentage of each type of milk including the final solution

	Amount	Part	Total
Milk 1	x	0.24	$0.24x$
Milk 2	y	0.18	$0.18y$
Final	42	0.2	8.4

Multiply amount by part to get totals.

$$x+y=42$$ The amount column gives one equation

$$0.24x+0.18y=8.4$$ The total column gives a second equation.

$$-0.18(x+y)=(42)(-0.18)$$ Use addition. Multiply first equation by -0.18

$$-0.18x-0.18y=-7.56$$

$-0.18x-0.18y=-7.56$	Add the equations together
$\underline{0.24x+0.18y=8.4}$	
$\frac{0.06x}{0.06}=\frac{0.84}{0.06}$	Divide both sides by 0.06
$x=14$	We have our x, 14 gal of 24% butterfat
$(14)+y=42$	Plug into original equation to find y
$\underline{-14\quad -14}$	Subtract 14 from both sides
$y=28$	We have our y, 28 gal of 18% butterfat
14 gal of 24% and 28 gal of 18%	Our Solution

The same process can be used to solve mixtures of prices with two unknowns.

Example 4.

In a candy shop, chocolate which sells for \$4 a pound is mixed with nuts which are sold for \$2.50 a pound are mixed to form a chocolate-nut candy which sells for \$3.50 a pound. How much of each are used to make 30 pounds of the mixture?

	Amount	Part	Total
Chocolate	c	4	
Nut	n	2.5	
Final	30	3.5	

Using our mixture table, use c and n for variables
We do know the final amount (30) and price, include this in the table

	Amount	Part	Total
Chocolate	c	4	$4c$
Nut	n	2.5	$2.5n$
Final	30	3.5	105

Multiply amount by part to get totals

$c+n=30$	First equation comes from the first column
$4c+2.5n=105$	Second equation comes from the total column
$c+n=30$	We will solve this problem with substitution
$-n\quad -n$	Solve for c by subtracting n from the first equation
$c=30-n$	
$4(30-n)+2.5n=105$	Substitute into untouched equation
$120-4n+2.5n=105$	Distribute

$120 - 1.5n = 105$	Combine like terms
$\underline{-120 \qquad -120}$	Subtract 120 from both sides
$\frac{-1.5n}{-1.5} = \frac{-15}{-1.5}$	Divide both sides by -1.5
$n = 10$	We have our n, 10 lbs of nuts
$c = 30 - (10)$	Plug into $c =$ equation to find c
$c = 20$	We have our c, 20 lbs of chocolate
10 lbs of nuts and 20 lbs of chocolate	Our Solution

With mixture problems we often are mixing with a pure solution or using water which contains none of our chemical we are interested in. For pure solutions, the percentage is 100% (or 1 in the table). For water, the percentage is 0%. This is shown in the following example.

Example 5.

A solution of pure antifreeze is mixed with water to make a 65% antifreeze solution. How much of each should be used to make 70 L?

	Amount	Part	Final
Antifreeze	a	1	
Water	w	0	
Final	70	0.65	

We use a and w for our variables. Antifreeze is pure, 100% or 1 in our table, written as a decimal. Water has no antifreeze, its percentage is 0. We also fill in the final percent

	Amount	Part	Final
Antifreeze	a	1	a
Water	w	0	0
Final	70	0.65	45.5

Multiply to find final amounts

$a + w = 70$	First equation comes from first column
$a = 45.5$	Second equation comes from second column
$(45.5) + w = 70$	We have a, plug into to other equation
$-45.5 \qquad -45.5$	Subtract 45.5 from both sides
$w = 24.5$	We have our w
$45.5\,L$ of antifreeze and $24.5\,L$ of water	Our Solution

4.6 Practice - Mixture Problems

Solve.

1) A tank contains 8000 liters of a solution that is 40% acid. How much water should be added to make a solution that is 30% acid?

2) How much antifreeze should be added to 5 quarts of a 30% mixture of antifreeze to make a solution that is 50% antifreeze?

3) Of 12 pounds of salt water 10% is salt; of another mixture 3% is salt. How many pounds of the second should be added to the first in order to get a mixture of 5% salt?

4) How much alcohol must be added to 24 gallons of a 14% solution of alcohol in order to produce a 20% solution?

5) How many pounds of a 4% solution of borax must be added to 24 pounds of a 12% solution of borax to obtain a 10% solution of borax?

6) How many grams of pure acid must be added to 40 grams of a 20% acid solution to make a solution which is 36% acid?

7) A 100 LB bag of animal feed is 40% oats. How many pounds of oats must be added to this feed to produce a mixture which is 50% oats?

8) A 20 oz alloy of platinum that costs $220 per ounce is mixed with an alloy that costs $400 per ounce. How many ounces of the $400 alloy should be used to make an alloy that costs $300 per ounce?

9) How many pounds of tea that cost $4.20 per pound must be mixed with 12 lb of tea that cost $2.25 per pound to make a mixture that costs $3.40 per pound?

10) How many liters of a solvent that costs $80 per liter must be mixed with 6 L of a solvent that costs $25 per liter to make a solvent that costs $36 per liter?

11) How many kilograms of hard candy that cost $7.50 per kilogram must be mixed with 24 kg of jelly beans that cost $3.25 per kilogram to make a mixture that sells for $4.50 per kilogram?

12) How many kilograms of soil supplement that costs $7.00 per kilogram must be mixed with 20 kg of aluminum nitrate that costs $3.50 per kilogram to make a fertilizer that costs $4.50 per kilogram?

13) How many pounds of lima beans that cost 90¢ per pound must be mixed with 16 lb of corn that cost 50¢ per pound to make a mixture of vegetables that costs 65¢ per pound?

14) How many liters of a blue dye that costs $1.60 per liter must be mixed with 18 L of anil that costs $2.50 per liter to make a mixture that costs $1.90 per liter?

15) Solution A is 50% acid and solution B is 80% acid. How much of each should be used to make 100cc. of a solution that is 68% acid?

16) A certain grade of milk contains 10% butter fat and a certain grade of cream

60% butter fat. How many quarts of each must be taken so as to obtain a mixture of 100 quarts that will be 45% butter fat?

17) A farmer has some cream which is 21% butterfat and some which is 15% butter fat. How many gallons of each must be mixed to produce 60 gallons of cream which is 19% butterfat?

18) A syrup manufacturer has some pure maple syrup and some which is 85% maple syrup. How many liters of each should be mixed to make 150L which is 96% maple syrup?

19) A chemist wants to make 50ml of a 16% acid solution by mixing a 13% acid solution and an 18% acid solution. How many milliliters of each solution should the chemist use?

20) A hair dye is made by blending 7% hydrogen peroxide solution and a 4% hydrogen peroxide solution. How many mililiters of each are used to make a 300 ml solution that is 5% hydrogen peroxide?

21) A paint that contains 21% green dye is mixed with a paint that contains 15% green dye. How many gallons of each must be used to make 60 gal of paint that is 19% green dye?

22) A candy mix sells for $2.20 per kilogram. It contains chocolates worth $1.80 per kilogram and other candy worth $3.00 per kilogram. How much of each are in 15 kilograms of the mixture?

23) To make a weed and feed mixture, the Green Thumb Garden Shop mixes fertilizer worth $4.00/lb. with a weed killer worth $8.00/lb. The mixture will cost $6.00/lb. How much of each should be used to prepare 500 lb. of the mixture?

24) A grocer is mixing 40 cent per lb. coffee with 60 cent per lb. coffee to make a mixture worth 54¢ per lb. How much of each kind of coffee should be used to make 70 lb. of the mixture?

25) A grocer wishes to mix sugar at 9 cents per pound with sugar at 6 cents per pound to make 60 pounds at 7 cents per pound. What quantity of each must he take?

26) A high-protein diet supplement that costs $6.75 per pound is mixed with a vitamin supplement that costs $3.25 per pound. How many pounds of each should be used to make 5 lb of a mixture that costs $4.65 per pound?

27) A goldsmith combined an alloy that costs $4.30 per ounce with an alloy that costs $1.80 per ounce. How many ounces of each were used to make a mixture of 200 oz costing $2.50 per ounce?

28) A grocery store offers a cheese and fruit sampler that combines cheddar cheese that costs $8 per kilogram with kiwis that cost $3 per kilogram. How many kilograms of each were used to make a 5 kg mixture that costs $4.50 per kilogram?

29) The manager of a garden shop mixes grass seed that is 60% rye grass with 70 lb of grass seed that is 80% rye grass to make a mixture that is 74% rye grass. How much of the 60% mixture is used?

30) How many ounces of water evaporated from 50 oz of a 12% salt solution to produce a 15% salt solution?

31) A caterer made an ice cream punch by combining fruit juice that cost $2.25 per gallon with ice cream that costs $3.25 per gallon. How many gallons of each were used to make 100 gal of punch costing $2.50 per pound?

32) A clothing manufacturer has some pure silk thread and some thread that is 85% silk. How many kilograms of each must be woven together to make 75 kg of cloth that is 96% silk?

33) A carpet manufacturer blends two fibers, one 20% wool and the second 50% wool. How many pounds of each fiber should be woven together to produce 600 lb of a fabric that is 28% wool?

34) How many pounds of coffee that is 40% java beans must be mixed with 80 lb of coffee that is 30% java beans to make a coffee blend that is 32% java beans?

35) The manager of a specialty food store combined almonds that cost $4.50 per pound with walnuts that cost $2.50 per pound. How many pounds of each were used to make a 100 lb mixture that cost $3.24 per pound?

36) A tea that is 20% jasmine is blended with a tea that is 15% jasmine. How many pounds of each tea are used to make 5 lb of tea that is 18% jasmine?

37) How many ounces of dried apricots must be added to 18 oz of a snack mix that contains 20% dried apricots to make a mixture that is 25% dried apricots?

38) How many mililiters of pure chocolate must be added to 150 ml of chocolate topping that is 50% chocolate to make a topping that is 75% chocolate?

39) How many ounces of pure bran flakes must be added to 50 oz of cereal that is 40% bran flakes to produce a mixture that is 50% bran flakes?

40) A ground meat mixture is formed by combining meat that costs $2.20 per pound with meat that costs $4.20 per pound. How many pounds of each were used to make a 50 lb mixture tha costs $3.00 per pound?

41) How many grams of pure water must be added to 50 g of pure acid to make a solution that is 40% acid?

42) A lumber company combined oak wood chips that cost $3.10 per pound with pine wood chips that cost $2.50 per pound. How many pounds of each were used to make an 80 lb mixture costing $2.65 per pound?

43) How many ounces of pure water must be added to 50 oz of a 15% saline solution to make a saline solution that is 10% salt?

4.6

Answers - Mixture Problems

1) 2666.7

2) 2

3) 30

4) 1, 8

5) 8

6) 10

7) 20

8) 16

9) 17.25

10) 1.5

11) 10

12) 8

13) 9.6

14) 36

15) 40, 60

16) 30, 70

17) 40, 20

18) 40, 110

19) 20, 30

20) 100, 200

21) 40, 20

22) 10, 5

23) 250, 250

24) 21, 49

25) 20, 40

26) 2, 3

27) 56, 144

28) 1.5, 3.5

29) 30

30) 10

31) 75, 25

32) 55, 20

33) 440, 160

34) 20

35) 35, 63

36) 3, 2

37) 1.2

38) 150

39) 10

40) 30, 20

41) 75

42) 20, 60

43) 25