Quadratics - Revenue and Distance

Objective: Solve revenue and distance applications of quadratic equations.

A common application of quadratics comes from revenue and distance problems. Both are set up almost identical to each other so they are both included together. Once they are set up, we will solve them in exactly the same way we solved the simultaneous product equations.

Revenue problems are problems where a person buys a certain number of items for a certain price per item. If we multiply the number of items by the price per item we will get the total paid. To help us organize our information we will use the following table for revenue problems

	Number	Price	Total
First			
Second			

The price column will be used for the individual prices, the total column is used for the total paid, which is calculated by multiplying the number by the price. Once we have the table filled out we will have our equations which we can solve. This is shown in the following examples.

Example 1.

A man buys several fish for $56. After three fish die, he decides to sell the rest at a profit of $5 per fish. His total profit was $4. How many fish did he buy to begin with?

	Number	Price	Total
Buy	n	p	56
Sell			

Using our table, we don't know the number he bought, or at what price, so we use varibles n and p. Total price was $56.

	Number	Price	Total
Buy	n	p	56
Sell	$n-3$	$p+5$	60

When he sold, he sold 3 less $(n-3)$, for $5 more $(p+5)$. Total profit was $4, combined with $56 spent is $60

$$np = 56 \quad \text{Find equatinos by multiplying number by price}$$

$$(n-3)(p+5) = 60 \quad \text{These are a simultaneous product}$$

$$p = \frac{56}{n} \ \text{ and } \ p + 5 = \frac{60}{n-3} \quad \text{Solving for number, divide by } n \text{ or } (n-3)$$

$$\frac{56}{n}+5=\frac{60}{n-3} \qquad \text{Substitute } \frac{56}{n} \text{ for } p \text{ in second equation}$$

$$\frac{56n(n-3)}{n}+5n(n-3)=\frac{60n(n-3)}{n-3} \qquad \text{Multiply each term by LCD: } n(n-3)$$

$$56(n-3)+5n(n-3)=60n \qquad \text{Reduce fractions}$$
$$56n-168+5n^2-15n=60n \qquad \text{Combine like terms}$$
$$5n^2+41n-168=60n \qquad \text{Move all terms to one side}$$
$$\underline{-60n \qquad -60n}$$
$$5n^2-19n-168=0 \qquad \text{Solve with quadratic formula}$$
$$n=\frac{19\pm\sqrt{(-19)^2-4(5)(-168)}}{2(5)} \qquad \text{Simplify}$$

$$n=\frac{19\pm\sqrt{3721}}{10}=\frac{19\pm 61}{10} \qquad \text{We don't want negative solutions, only do } +$$

$$n=\frac{80}{10}=8 \qquad \text{This is our } n$$

8 fish   Our Solution

Example 2.

A group of students together bought a couch for their dorm that cost \$96. However, 2 students failed to pay their share, so each student had to pay \$4 more. How many students were in the original group?

	Number	Price	Total
Deal	n	p	96
Paid			

\$96 was paid, but we don't know the number or the price agreed upon by each student.

	Number	Price	Total
Deal	n	p	96
Paid	$n-2$	$p+4$	96

There were 2 less that actually paid $(n-2)$ and they had to pay \$4 more $(p+4)$. The total here is still \$96.

$$np=96 \qquad \text{Equations are product of number and price}$$
$$(n-2)(p+4)=96 \qquad \text{This is a simultaneous product}$$

$$p=\frac{96}{n} \text{ and } p+4=\frac{96}{n-2} \qquad \text{Solving for number, divide by } n \text{ and } n-2$$

$$\frac{96}{n}+4=\frac{96}{n-2} \qquad \text{Substitute } \frac{96}{n} \text{ for } p \text{ in the second equation}$$

$$\frac{96n(n-2)}{n} + 4n(n-2) = \frac{96n(n-2)}{n-2} \quad \text{Multiply each term by LCD: } n(n-2)$$

$$\begin{aligned}
96(n-2) + 4n(n-2) &= 96n && \text{Reduce fractions} \\
96n - 192 + 4n^2 - 8n &= 96n && \text{Distribute} \\
4n^2 + 88n - 192 &= 96n && \text{Combine like terms} \\
\underline{-96n \qquad\quad -96n} & && \text{Set equation equal to zero} \\
4n^2 - 8n - 192 &= 0 && \text{Solve by completing the square,} \\
\underline{+192 + 192} & && \text{Separate variables and constant} \\
4n^2 - 8n &= 192 && \text{Divide each term by } a \text{ or } 4 \\
\overline{4} \quad \overline{4} \quad \overline{4} & && \\
n^2 - 2n &= 48 && \text{Complete the square: } \left(b \cdot \frac{1}{2}\right)^2 \\
\left(2 \cdot \frac{1}{2}\right)^2 = 1^2 &= 1 && \text{Add to both sides of equation} \\
n^2 - 2n + 1 &= 49 && \text{Factor} \\
(n-1)^2 &= 49 && \text{Square root of both sides} \\
n - 1 &= \pm 7 && \text{Add 1 to both sides} \\
n &= 1 \pm 7 && \text{We don't want a negative solution} \\
n &= 1 + 7 = 8 && \\
8 \text{ students} & && \text{Our Solution}
\end{aligned}$$

The above examples were solved by the quadratic formula and completing the square. For either of these we could have used either method or even factoring. Remember we have several options for solving quadratics. Use the one that seems easiest for the problem.

Distance problems work with the same ideas that the revenue problems work. The only difference is the variables are r and t (for rate and time), instead of n and p (for number and price). We already know that distance is calculated by multiplying rate by time. So for our distance problems our table becomes the following:

	rate	time	distance
First			
Second			

Using this table and the exact same patterns as the revenue problems is shown in the following example.

Example 3.

Greg went to a conference in a city 120 miles away. On the way back, due to road construction he had to drive 10 mph slower which resulted in the return trip taking 2 hours longer. How fast did he drive on the way to the conference?

	rate	time	distance
There	r	t	120
Back			

We do not know rate, r, or time, t he traveled on the way to the conference. But we do know the distance was 120 miles.

	rate	time	distance
There	r	t	120
Back	$r-10$	$t+2$	120

Coming back he drove 10 mph slower $(r-10)$ and took 2 hours longer $(t+2)$. The distance was still 120 miles.

$$rt=120 \qquad \text{Equations are product of rate and time}$$

$$(r-10)(t+2)=120 \qquad \text{We have simultaneous product equations}$$

$$t=\frac{120}{r} \text{ and } t+2=\frac{120}{r-10} \qquad \text{Solving for rate, divide by } r \text{ and } r-10$$

$$\frac{120}{r}+2=\frac{120}{r-10} \qquad \text{Substitute } \frac{120}{r} \text{ for } t \text{ in the second equation}$$

$$\frac{120r(r-10)}{r}+2r(r-10)=\frac{120r(r-10)}{r-10} \qquad \text{Multiply each term by LCD: } r(r-10)$$

$$120(r-10)+2r^2-20r=120r \qquad \text{Reduce each fraction}$$

$$120r-1200+2r^2-20r=120r \qquad \text{Distribute}$$

$$2r^2+100r-1200=120r \qquad \text{Combine like terms}$$

$$\underline{-120r \qquad\qquad -120r} \qquad \text{Make equation equal to zero}$$

$$2r^2-20r-1200=0 \qquad \text{Divide each term by 2}$$

$$r^2-10r-600=0 \qquad \text{Factor}$$

$$(r-30)(r+20)=0 \qquad \text{Set each factor equal to zero}$$

$$r-30=0 \text{ and } r+20=0 \qquad \text{Solve each equation}$$

$$\underline{+30+30} \qquad \underline{-20-20}$$

$$r=30 \text{ and } r=-20 \qquad \text{Can't have a negative rate}$$

$$30\text{ mph} \qquad \text{Our Solution}$$

World View Note: The world's fastest man (at the time of printing) is Jamaican Usain Bolt who set the record of running 100 m in 9.58 seconds on August 16, 2009 in Berlin. That is a speed of over 23 miles per hour!

Another type of simultaneous product distance problem is where a boat is traveling in a river with the current or against the current (or an airplane flying with the wind or against the wind). If a boat is traveling downstream, the current will push it or increase the rate by the speed of the current. If a boat is traveling upstream, the current will pull against it or decrease the rate by the speed of the current. This is demonstrated in the following example.

Example 4.

A man rows down stream for 30 miles then turns around and returns to his original location, the total trip took 8 hours. If the current flows at 2 miles per hour, how fast would the man row in still water?

8 — Write total time above time column

	rate	time	distance
down		t	30
up		$8-t$	30

We know the distance up and down is 30. Put t for time downstream. Subtracting $8-t$ becomes time upstream

	rate	time	distance
down	$r+2$	t	30
up	$r-2$	$8-t$	30

Downstream the current of 2 mph pushes the boat $(r+2)$ and upstream the current pulls the boat $(r-2)$

$$(r+2)t=30 \quad \text{Multiply rate by time to get equations}$$
$$(r-2)(8-t)=30 \quad \text{We have a simultaneous product}$$

$$t=\frac{30}{r+2} \text{ and } 8-t=\frac{30}{r-2} \quad \text{Solving for rate, divide by } r+2 \text{ or } r-2$$

$$8-\frac{30}{r+2}=\frac{30}{r-2} \quad \text{Substitute } \frac{30}{r+2} \text{ for } t \text{ in second equation}$$

$$8(r+2)(r-2)-\frac{30(r+2)(r-2)}{r+2}=\frac{30(r+2)(r-2)}{r-2} \quad \text{Multiply each term by LCD: } (r+2)(r-2)$$

$$8(r+2)(r-2)-30(r-2)=30(r+2) \quad \text{Reduce fractions}$$
$$8r^2-32-30r+60=30r+60 \quad \text{Multiply and distribute}$$
$$8r^2-30r+28=30r+60 \quad \text{Make equation equal zero}$$
$$\underline{-30r-60-30r-60}$$
$$8r^2-60r-32=0 \quad \text{Divide each term by 4}$$
$$2r^2-15r-8=0 \quad \text{Factor}$$
$$(2r+1)(r-8)=0 \quad \text{Set each factor equal to zero}$$
$$2r+1=0 \text{ or } r-8=0 \quad \text{Solve each equation}$$
$$\underline{-1-1} \qquad \underline{+8+8}$$
$$\frac{2r}{2}=\frac{-1}{2} \text{ or } r=8$$
$$r=-\frac{1}{2} \text{ or } r=8 \quad \text{Can't have a negative rate}$$
$$8 \text{ mph} \quad \text{Our Solution}$$

9.10 Practice - Revenue and Distance

1) A merchant bought some pieces of silk for \$900. Had he bought 3 pieces more for the same money, he would have paid \$15 less for each piece. Find the number of pieces purchased.

2) A number of men subscribed a certain amount to make up a deficit of \$100 but 5 men failed to pay and thus increased the share of the others by \$1 each. Find the amount that each man paid.

3) A merchant bought a number of barrels of apples for \$120. He kept two barrels and sold the remainder at a profit of \$2 per barrel making a total profit of \$34. How many barrels did he originally buy?

4) A dealer bought a number of sheep for \$440. After 5 had died he sold the remainder at a profit of \$2 each making a profit of \$60 for the sheep. How many sheep did he originally purchase?

5) A man bought a number of articles at equal cost for \$500. He sold all but two for \$540 at a profit of \$5 for each item. How many articles did he buy?

6) A clothier bought a lot of suits for \$750. He sold all but 3 of them for \$864 making a profit of \$7 on each suit sold. How many suits did he buy?

7) A group of boys bought a boat for \$450. Five boys failed to pay their share, hence each remaining boys were compelled to pay \$4.50 more. How many boys were in the original group and how much had each agreed to pay?

8) The total expenses of a camping party were \$72. If there had been 3 fewer persons in the party, it would have cost each person \$2 more than it did. How many people were in the party and how much did it cost each one?

9) A factory tests the road performance of new model cars by driving them at two different rates of speed for at least 100 kilometers at each rate. The speed rates range from 50 to 70 km/hr in the lower range and from 70 to 90 km/hr in the higher range. A driver plans to test a car on an available speedway by driving it for 120 kilometers at a speed in the lower range and then driving 120 kilometers at a rate that is 20 km/hr faster. At what rates should he drive if he plans to complete the test in $3\frac{1}{2}$ hours?

10) A train traveled 240 kilometers at a certain speed. When the engine was replaced by an improved model, the speed was increased by 20 km/hr and the travel time for the trip was decreased by 1 hour. What was the rate of each engine?

11) The rate of the current in a stream is 3 km/hr. A man rowed upstream for 3 kilometers and then returned. The round trip required 1 hour and 20 minutes. How fast was he rowing?

12) A pilot flying at a constant rate against a headwind of 50 km/hr flew for 750 kilometers, then reversed direction and returned to his starting point. He completed the round trip in 8 hours. What was the speed of the plane?

13) Two drivers are testing the same model car at speeds that differ by 20 km/hr. The one driving at the slower rate drives 70 kilometers down a speedway and returns by the same route. The one driving at the faster rate drives 76 kilometers down the speedway and returns by the same route. Both drivers leave at the same time, and the faster car returns $\frac{1}{2}$ hour earlier than the slower car. At what rates were the cars driven?

14) An athlete plans to row upstream a distance of 2 kilometers and then return to his starting point in a total time of 2 hours and 20 minutes. If the rate of the current is 2 km/hr, how fast should he row?

15) An automobile goes to a place 72 miles away and then returns, the round trip occupying 9 hours. His speed in returning is 12 miles per hour faster than his speed in going. Find the rate of speed in both going and returning.

16) An automobile made a trip of 120 miles and then returned, the round trip occupying 7 hours. Returning, the rate was increased 10 miles an hour. Find the rate of each.

17) The rate of a stream is 3 miles an hour. If a crew rows downstream for a distance of 8 miles and then back again, the round trip occupying 5 hours, what is the rate of the crew in still water?

18) The railroad distance between two towns is 240 miles. If the speed of a train were increased 4 miles an hour, the trip would take 40 minutes less. What is the usual rate of the train?

19) By going 15 miles per hour faster, a train would have required 1 hour less to travel 180 miles. How fast did it travel?

20) Mr. Jones visits his grandmother who lives 100 miles away on a regular basis. Recently a new freeway has opend up and, although the freeway route is 120 miles, he can drive 20 mph faster on average and takes 30 minutes less time to make the trip. What is Mr. Jones rate on both the old route and on the freeway?

21) If a train had traveled 5 miles an hour faster, it would have needed $1\frac{1}{2}$ hours less time to travel 150 miles. Find the rate of the train.

22) A traveler having 18 miles to go, calculates that his usual rate would make him one-half hour late for an appointment; he finds that in order to arrive on time he must travel at a rate one-half mile an hour faster. What is his usual rate?

Answers - Revenue and Distance

1) 12
2) \$4
3) 24
4) 55
5) 20
6) 30
7) 25 @ \$18
8) 12 @ \$6
9) 60 mph, 80 mph
10) 60, 80
11) 6 km/hr
12) 200 km/hr
13) 56, 76
14) 3.033 km/hr
15) 12 mph, 24 mph
16) 30 mph, 40 mph
17) $r = 5$
18) 36 mph
19) 45 mph
20) 40 mph, 60 mph
21) 20 mph
22) 4 mph