4.4

1)
$(I) \quad a - 2b + c = 5$
$(II)2a + b - c = -1$
$(III)3a + 3b - 2c = -4$

$A: -1(3a - b) = 4(-1)$
$B: 5a - b = 6$
$\underline{-3a + b = -4}$
$\frac{2a}{2} = \frac{2}{2}$
$a = 1$

$(I) \quad a - 2b + c = 5$
$(II)\underline{2a + b - c = -1}$
$A: 3a - b = 4$

$A: 3(1) - b = 4$
$3 - b = 4$
$\underline{-3 \qquad -3}$
$\frac{-b}{-1} = \frac{1}{-1}$
$b = -1$

$(I) 2(a - 2b + c) = (5)2$
$(III)3a + 3b - 2c = -4$
$\underline{2a - 4b + 2c = 10}$
$B: 5a - b = 6$

$(I) (1) - 2(-1) + c = 5$
$1 + 2 + c = 5$
$3 + c = 5$
$\underline{-3 \qquad -3}$
$c = 2$

$(1, -1, 2)$

3)
$(I) \ 3x + y - z = 11$
$(II) \ x + 3y = z + 13$
$(III) \ x + y - 3z = 11$

$(II)x + 3y = z + 13$
$\underline{-z - z}$
$(II) \ x + 3y - z = 13$

$(I) -1(3x + y - z) = 11(-1)$
$(II) \qquad x + 3y - z = 13$
$\underline{-3x - y + z = -11}$
$A: -2x + 2y = 2$

$(I) -3(3x + y - z) = 11(-3)$
$(III) \qquad x + y - 3z = 11$
$\underline{-9x - 3y + 3z = -33}$
$B: -8x - 2y = -22$

$A: -2x + 2y = 2$
$B: \underline{-8x - 2y = -22}$
$-\frac{10x}{-10} = -\frac{20}{-10}$
$x = 2$

$(2, 3, -2)$

$A: -2(2) + 2y = 2$
$-4 + 2y = 2$
$\underline{+4 \qquad +4}$
$\frac{2y}{2} = \frac{6}{2}$
$y = 3$

$(I) \ 3(2) + 3 - z = 11$
$6 + 3 - z = 11$
$9 - z = 11$
$\underline{-9 \qquad -9}$
$\frac{-z}{-1} = \frac{2}{-1}$
$z = -2$

5)
$(I)x + 6y + 3z = 4$
$(II) \ 2x + y + 2z = 3$
$(III)3x - 2y + z = 0$

$(I) \ x + 6y + 3z = 4$
$(III) \ 3(3x - 2y + z) = (0)3$
$x + 6y + 3z = 4$
$\underline{9x - 6y + 3z = 0}$
$A: 10x + 6z = 4$

$(II)2(2x + y + 2z) = (3)2$
$(III) \ 3x - 2y + z = 0$
$\underline{4x + 2y + 4z = 6}$
$B: 7x + 5z = 6$

$A: -5(10x + 6z) = 4(-5)$
$B: 6(7x + 5z) = 6(6)$
$-50x - 30z = -20$
$\underline{42x + 30z = 36}$
$\frac{-8x}{-8} = \frac{16}{-8}$
$x = -2$

$A: 10(-2) + 6z = 4$
$-20 + 6z = 4$
$\underline{+20 \qquad +20}$
$\frac{6z}{6} = \frac{24}{6}$
$z = 4$

$(-2, -1, 4)$

$(I) \ (-2) + 6y + 3(4) = 4$
$-2 + 6y + 12 = 4$
$10 + 6y = 4$
$\underline{-10 \qquad -10}$
$\frac{6y}{6} = \frac{-6}{6}$
$y = -1$

7) (I) $x + y + z = 6$

$(II)2x - y - z = -3$

(III) $x - 2y + 3z = 6$

$A: 3(-y - z) = (-5)3$

$B: -2y + 3z = 5$

$\underline{-3y - 3z = -15}$

$-\dfrac{5y}{-5} = -\dfrac{10}{-5}$

$y = 2$

(I) $x + y + z = 6$

$\underline{(II)2x - y - z = -3}$

$\dfrac{3x}{3} = \dfrac{3}{3}$

$x = 1$

$A: -(2) - z = -5$

$\underline{+2 \qquad\quad +2}$

$-\dfrac{z}{-1} = -\dfrac{3}{-1}$

$z = 3$

$(II)2(1) - y - z = -3$

$2 - y - z = -3$

$\underline{-2 \qquad\qquad -2}$

$A: \quad -y - z = -5$

$(III)1 - 2y + 3z = 6$

$\underline{-1 \qquad\qquad -1}$

$B: \ -2y + 3z = 5$

$(1, 2, 3)$

9) (I) $x + y - z = 0$

$(II)x - y - z = 0$

$(III)x + y + 2x = 0$

$A: (-1)(2x - 2z) = 0(-1)$

$B: 2x + z = 0$

$\underline{-2x + 2z = 0}$

$\dfrac{3z}{0} = 0$

$x = 0$

(I) $x + y - z = 0$

$\underline{(II)x - y - z = 0}$

$A: 2x - 2z = 0$

$A: 2x - 2(0) = 0$

$\dfrac{2x}{2} = \dfrac{0}{2}$

$x = 0$

$(II)x - y - z = 0$

$\underline{(III)\ x + y + 2z = 0}$

$B: 2x + z = 0$

$(I)\ 0 + y - 0 = 0$

$y = 0$

$(0, 0, 0)$

11) $(I) -2x + y - 3z = 1$

$(II)\ \ x - 4y + z = 6$

$(III)4x + 16y + 4z = 24$

$A: -2(-7y - z) = 13(-2)$

$B: 18y - 2z = 26$

$\underline{14y + 2z = -26}$

$\dfrac{32y}{32} = \dfrac{0}{32}$

$y = 0$

$(I) -2x + y - 3z = 1$

$(II)2(x - 4y + z) = 6(2)$

$-2x + y - 3z = 1$

$\underline{2x - 8y + 2z = 12}$

$A: \qquad -7y - z = 13$

$A: -7(0) - z = 13$

$-\dfrac{z}{-1} = \dfrac{13}{-1}$

$z = -13$

$(19, 0, -13)$

$(I)2(-2x + y - 3z) = (1)2$

$(III)4x + 16y + 4z = 24$

$\underline{-4x + 2y - 6z = 2}$

$B: 18y - 2z = 26$

$(I) -2x + 0 - 3(-13) = 1$

$-2x + 39 = 1$

$\underline{-39 \ -39}$

$-\dfrac{2x}{-2} = -\dfrac{38}{-2}$

$x = 19$

13)

$(I)\,2x + y - 3z = 0$
$(II)\,x - 4y + z = 0$
$(III)\,4x + 16y + 4z = 0$

$A:8(9x - 11z) = 0(8)$
$B:-9(8x + 8z) = 2(-9)$
$\quad 72x - 88z = 0$
$\quad \underline{-72x - 72z = 0}$
$\quad\quad \frac{-160z}{-160} = 0$
$\quad\quad\quad z = 0$

$(I)\,4(2x + y - 3z) = (0)4$
$(II)\ \ x - 4y + z = 0$
$\quad \underline{-8x + 4y - 12z = 0}$
$\quad A:9x - 11z = 0$

$A:9x - 11(0) = 0$
$\quad \frac{9x}{9} = \frac{0}{9}$
$\quad x = 0$

$(II)\,4(x - 4y + z) = (0)4$
$(III)\,4x + 16y + 4z = 0$
$\quad \underline{4x - 16y + 4z = 0}$
$B:\ \ 8x + 8z = 2$

$(I)\,2(0) + y - 3(0) = 0$
$\quad\quad\quad y = 0$

$\quad\quad (0, 0, 0)$

15)

$(I)\ 3x + 2y + 2x = 3$
$(II)\ x + 2y - z = 5$
$(III)\,2x - 4y + z = 0$

$A:3(3x - 2y) = (5)5$
$B:5x + 6y = 13$
$\quad \underline{9x - 6y = 15}$
$\quad\quad \frac{14x}{14} = \frac{28}{14}$
$\quad\quad\quad x = 2$

$\quad\quad (2, \tfrac{1}{2}, -2)$

$(II)\ x + 2y - z = 5$
$\underline{(III)\ 2x - 4y + z = 0}$
$A:3x - 2y = 5$

$A:3(2) - 2y = 5$
$\quad 6 - 2y = 5$
$\quad \underline{-6 \quad\quad\ -6}$
$\quad -\frac{2y}{-2} = -\frac{1}{-2}$
$\quad\quad y = \frac{1}{2}$

$(I)\ 3x + 2y + 2x = 3$
$(II)\ 2(x + 2y - z) = (5)2$
$\quad 3x + 2y + 2z = 3$
$\quad \underline{2x + 4y - 2z = 10}$
$B:5x + 6x = 13$

$(I)\ 3(2) + 2\left(\tfrac{1}{2}\right) + 2z = 3$
$\quad 6 + 1 + 2z = 3$
$\quad 7 + 2z = 3$
$\quad \underline{-7 \quad\quad\ -7}$
$\quad\quad \frac{2z}{2} = -\frac{4}{2}$
$\quad\quad z = -2$

17)

$(I)\,x - 2y + 3z = 4$
$(II)\ 2x - y + z = -1$
$(III)\ 4x + y + z = 1$

$A:3(6x + 2y) = (0)3$
$B:-2(9x + 5z) = 6(-2)$
$\quad 18x + 6z = 0$
$\quad \underline{-18x - 10z = -12}$
$\quad\quad -\frac{4z}{-4} = -\frac{12}{-4}$
$\quad\quad\quad z = 3$

$\quad\quad (-1, 2, 3)$

$(II)\ 2x - y + z = -1$
$\underline{(III)\,4x + y + z = 1}$
$A:6x + 2z = 0$

$A:6x + 2(3) = 0$
$\quad 6x + 6 = 0$
$\quad \underline{-6 \ -6}$
$\quad \frac{6x}{6} = \frac{-6}{6}$
$\quad\quad x = -1$

$(I)\ x - 2y + 3z = 4$
$(III)\ 2(4x + y + z) = (1)2$
$\quad x - 2y + 3z = 4$
$\quad \underline{8x + 2y + 2z = 2}$
$B:9x + 5z = 6$

$(I)\ (-1) - 2y + 3(3) = 4$
$\quad 8 - 2y = 4$
$\quad \underline{-8 \quad\quad\ -8}$
$\quad\quad \frac{-2y}{-2} = \frac{-4}{-2}$
$\quad\quad\quad y = 2$

19) $(I)\ x - y + 2z = 0$

$(II)x - 2y + 3z = -1$

$(III)2x - 2y + z = -3$

$(I)x - (2) + 2(1) = 0$

$x - 2 + 2 = 0$

$x = 0$

$(I)(-1)(x - y + 2z) = 0(-1)$

$(II)x - 2y + 3z = -1$

$\underline{-x + y - 2z = 0}$

$A{:}\ -y + z = -1$

$-y + (1) = -1$

$\underline{-1\quad -1}$

$-\dfrac{y}{-1} = -\dfrac{2}{-1}$

$y = 2$

$(I)\ (-2)(x - y + 2z) = 0(-2)$

$(III)2x - 2y + z = -3$

$\underline{-2x + 2y - 4z = 0}$

$-\dfrac{3z}{-3} = -\dfrac{3}{-3}$

$z = 1$

$(0, 2, 1)$

21) $(I)\ 4x - 3y + 2z = 40$

$(II)\ 5x + 9y - 7z = 47$

$(III)9x + 8y - 3z = 97$

$A{:}\ 7(17x - z) = (167)7$

$B{:}59x + 7z = 611$

$\underline{119x - 7z = 1169}$

$\dfrac{178x}{178} = \dfrac{1780}{178}$

$x = 10$

$(I)3(4x - 3y + 2z) = (40)3$

$(II)5x + 9y - 7z = 47$

$\underline{12x - 9y + 6z = 120}$

$A{:}17x - z = 167$

$A{:}17(10) - z = 167$

$170 - z = 167$

$\underline{-170\quad -170}$

$-\dfrac{z}{-1} = -\dfrac{3}{-1}$

$z = 3$

$(I)\ 8(4x - 3y + 2z) = (40)8$

$(III)3(9x + 8y - 3z) = (97)3$

$32x - 24y + 16z = 320$

$\underline{27x + 24y - 9z = 291}$

$B{:}59x + 7z = 611$

$(I)\ 4(10) - 3y + 2(3) = 40$

$46 - 3y = 40$

$\underline{-46\quad -46}$

$-\dfrac{3y}{-3} = -\dfrac{6}{-3}$

$y = 2$

$(10, 2, 3)$

23) $(I)3x + 3y - 2z = 13$

$(II)\ 6x + 2y - 5z = 13$

$(III)5x - 2y - 5z = -1$

$A{:}19(11x - 10z) = (12)19$

$B{:}(-10)(21x - 19z) = 23(-10)$

$209x - 190z = 228$

$\underline{-210 + 190z = -230}$

$-\dfrac{x}{-1} = -\dfrac{2}{-1}$

$x = 2$

$(2, 3, 1)$

$(II)6x + 2y - 5z = 13$

$(III)\underline{5x - 2y - 5z = -1}$

$A{:}11x - 10z = 12$

$A{:}11(2) - 10z = 12$

$22 - 10z = 12$

$\underline{-22\quad -22}$

$-\dfrac{10z}{-10} = -\dfrac{10}{-10}$

$z = 1$

$(I)2(3x + 3y - 2z) = (13)2$

$(III)3(5x - 2y - 5z) = (-1)3$

$6x + 6y - 4z = 26$

$\underline{15x - 6y - 15z = -3}$

$B{:}21x - 19z = 23$

$(I)\ 3(2) + 3y - 2(1) = 13$

$4 + 3y = 13$

$\underline{-4\quad -4}$

$\dfrac{3y}{3} = \dfrac{9}{3}$

$y = 3$

25) $(I)\ 3x - 4y + 2z = 1$

$(II)\ 2x + 3y - 3z = -1$

$(III)\ x + 10y - 8z = 7$

$A: -34y + 26z = -20$

$B: -2(-17y + 13z) = -15(-2)$

$\quad\ -34y + 26z = -20$

$\underline{\quad\ 34y - 26z = 30\quad}\qquad\ false$

$\qquad\qquad 0 = 10\qquad\quad No\ solution\ \theta$

$(I)\ 3x - 4y + 2z = 1$

$(III)\ (-3)(x + 10y - 8z) = 7(-3)$

$\quad\ 3x - 4y + 2z = 1$

$\underline{-3x - 30y + 24z = -21}$

$A: -34y + 26z = -20$

$(II)\ 2x + 3y - 3z = -1$

$(III)\ (-2)(x + 10y - 8z) = 7(-2)$

$\quad\ 2x + 3y - 3z = -1$

$\underline{-2x - 20y + 16z = -14}$

$B: -17y + 13z = -15$

27) $(I)\ m + 6n + 3p = 8$

$(II)\ 3m + 4n = -3$

$(III)\ 5m + 7n = 1$

$(II)\ (-5)(3m + 4n) = (-3)(-5)$

$(III)\ 3(5m + 7n) = (1)3$

$\quad\ -15m - 20n = 15$

$\underline{\quad\ 15m + 21n = 3\quad}$

$\qquad\qquad n = 18$

$(II)\ 3m + 4(18) = -3$

$\quad\ 3m + 72 = -3$

$\underline{\quad\ -72\quad\ -72}$

$\dfrac{3m}{3} = -\dfrac{75}{3}$

$\quad\ m = -25$

$(I)\ (-25) + 6(18) + 3p = 8$

$\quad\ -25 + 108 + 3p = 8$

$\qquad\quad 83 + 3p = 8$

$\underline{\quad\ -83\qquad -83}$

$\qquad\quad \dfrac{3p}{3} = -\dfrac{75}{3}$

$\qquad\qquad p = -25$

$(-25, 18, -25)$

29) $(I)\ -2w + 2x + 2y - 2z = -10$

$(II)\ w + x + y + z = -5$

$(III)\ 3w + 2x + 2y + 4z = -11$

$(IV)\ w + 3x - 2y + 2z = -6$

$(I)\ -2w + 2x + 2y - 2z = -10$

$(II)\ (-2)(w + x + y + z) = (-5)(-2)$

$\quad\ -2w + 2x + 2y - 2z = -10$

$\underline{\quad\ -2w - 2x - 2y - 2z = 10}$

$A: -4w - 4z = 0$

$(I)\ (-1)(-2w + 2x + 2y - 2z) = (-10)(-1)$

$(III)\ 3w + 2x + 2y + 4z = -11$

$\underline{\quad\ 2w - 2x - 2y + 2z = 10}$

$\quad\ B: 5w + 6z = -1$

$A: 3(-4w - 4z) = 0(3)$

$B: 2(5w + 6z) = (-1)2$

$\quad\ -12w - 12z = 0$

$\underline{\quad\ 10w + 12z = -2}$

$\qquad -\dfrac{2w}{-2} = -\dfrac{2}{-2}$

$\qquad\qquad w = 1$

$A: -4(1) - 4z = 0$

$\quad\ -4 - 4z = 0$

$\underline{\quad +4\qquad +4}$

$\quad -\dfrac{4z}{-4} = \dfrac{4}{-4}$

$\qquad\quad z = -1$

$(III)\ 3(1) + 2x + 2y + 4(-1) = -11$

$\qquad 3 + 2x + 2y - 4 = -11$

$\qquad 2x + 2y - 1 = -11$

$\qquad \underline{\qquad +1 \quad +1\qquad}$

$\qquad C: 2x + 2y = -10$

$(IV)\ (1) + 3x - 2y + 2(-1) = -6$

$\qquad 1 + 3x - 2y - 2 = -6$

$\qquad 3x - 2y - 1 = -6$

$\qquad \underline{\qquad +1 \quad +1\qquad}$

$\qquad D: 3x - 2y = -5$

$C: 2x + 2y = -10$

$D: 3x - 2y = -5$

$\dfrac{5x}{5} = -\dfrac{15}{5}$

$x = -3$

$C: 2(-3) + 2y = -10$

$\qquad -6 + 2y = -10$

$\qquad \underline{+6 \qquad \quad +6}$

$\qquad \dfrac{2y}{2} = -\dfrac{4}{2}$

$\qquad y = -2$

$(1, -3, -2, -1)$

31) $(I)\ w + x + y + z = 2$

$(II)\ w + 2x + 2y + 4z = 1$

$(III)\ -w + x - y - z = -2$

$(IV)\ -w + 3x + y - z = -2$

$(I)\ w + x + y + z = 2$

$(II\underline{I)\ -w + x - y - z = -6}$

$\qquad \dfrac{2x}{2} = -\dfrac{4}{2}$

$\qquad x = -2$

$(III)(-1)(-w + x - y - z) = (-6)(-1)$

$(IV)\ -w + 3x + y - z = -2$

$\underline{\qquad w - x + y + z = 6\qquad}$

$\qquad 2x + 2y = 4$

$\qquad 2(-2) + 2y = 4$

$\qquad -4 + 2y = 4$

$\qquad \underline{+4 \qquad \quad +4}$

$\qquad \dfrac{2y}{2} = \dfrac{8}{2}$

$\qquad y = 4$

$(II)\ w + 2x + 2y + 4z = 1$

$(IV\underline{)\ -2 + 3x + y - z = -2}$

$\qquad 5x + 3y + 3z = -1$

$\qquad 5(-2) + 3(4) + 3z = -1$

$\qquad -10 + 12 + 3z = -1$

$\qquad 2 + 3z = -1$

$\qquad \underline{-2 \qquad \quad -2}$

$\qquad \dfrac{3z}{3} = \dfrac{-3}{3}$

$\qquad z = -1$

$(I)\ w + (-2) + (4) + (-1) = 2$

$\qquad w + 1 = 2$

$\qquad \underline{-1 \ -1}$

$\qquad w = 1$

$(1, -2, 4, -1)$