9.10

1) A merchant bought some pieces of silk for $900. Had he bought 3 pieces more for the same money, he would have paid $15 less for each piece. Find the number of pieces purchased.

N	P	T
N	P	900
$n+3$	$p-15$	900

$\frac{np}{n} = \frac{900}{n} \quad p = 900n$

$\frac{(n+3)(p-15)}{n+3} = \frac{900}{n+3}$

$n + 3 = \frac{900}{n+3}$

$\frac{900}{n}\big(n(n+3)\big) - 15\big(n(n+3)\big) = \frac{900}{n+3}n(n+3)$

$LCD: n(n+3)$

$900(n+3) - 15n(n+3) = 900n$

$900n + 2700 - 15n^2 - 45n = 900n$

$-15n^2 + 855n + 2700 = 900n$

$+15n^2 - 855n - 2700 \quad +15n^2 - 855n - 2700$

$0 = \frac{15n^2}{15} + \frac{45n}{15} - \frac{2700}{15}$

$0 = n^2 + 3n - 180$

$0 = (n+15)(n-12)$

$n + 15 = 0 \quad n - 12 = 0$

$-15 \quad -15 \quad +12 \quad +12$

~~$n = -15$~~ $\quad n = 12$

3) A merchant bought a number of barrels of apples for S120. He kept two barrels and sold the remainder at a profit of $2 per barrel making a total profit of $34. How many barrels did he originally buy?

N	P	T
N	P	120
$n-2$	$p+2$	900

$\frac{np}{n} = \frac{120}{n} \quad p = \frac{120}{n}$

$\frac{(n-2)(p+2)}{n-2} = \frac{154}{n-2}$

$p + 2 = \frac{154}{n-2}$

$\frac{120}{n}\big(n(n-2)\big) + 2\big(n(n-2)\big) = \frac{154}{n-2}\big(n(n-2)\big)$

$LCD: \big(n(n-2)\big)$

$120(n-2) + 2n(n-2) = 154n$

$120n - 240 + 2n^2 - 4n = 154n$

$2n^2 + 116n - 240 = 154n$

$-154n \qquad -154n$

$\frac{2n^2}{2} - \frac{38n}{2} - \frac{240}{2} = \frac{0}{2}$

$n^2 - 17n - 120 = 0$

$(n-24)(n+5) = 0$

$n - 24 = 0 \quad n + 5 = 0$

$+24 \quad +24 \quad -5 \quad -5$

$n = 24 \quad$ ~~$n = -5$~~

5) A man bought a number of articles at equal cost for \$500. He sold all but two for \$540 at a profit of S5 for each item. How many articles did he buy?

N	P	T
N	P	500
$n-2$	$p+5$	540

$\frac{np}{n} = \frac{500}{n} \quad p = \frac{500}{n}$

$\frac{(n-2)(p+5)}{n-2} = \frac{540}{n-2}$

$p + 5 = \frac{540}{n-2}$

$\frac{500}{n}\big(n(n+2)\big) + 5\big(n(n+2)\big) = \frac{540}{n-2}\big(n(n+2)\big)$

$LCD: \big(n(n+2)\big)$

$500(n-2) + 5n(n-2) = 540n$

$500n - 1000 + 5n^2 - 10n = 540n$

$5n^2 + 490n - 1000 = 540n$

$-540n \qquad -540n$

$\frac{5n^2}{5} - \frac{50n}{5} - \frac{1000}{5} = \frac{0}{5}$

$n^2 - 10n - 200 = 0$

$(n-20)(n+40) = 0$

$n - 20 = 0 \quad n + 40 = 0$

$+20 \;\; +20 \quad -40 \;\; -40$

$n = 20 \qquad \cancel{n = -40}$

7) A group of boys bought a boat for \$450. Five boys failed to pay their share, hence each remaining boys were compelled to pay \$4.50 more. How many boys were in the original group and how much had each agreed to pay?

N	P	T
N	P	450
$n-5$	$p+4.5$	450

$\frac{np}{n} = \frac{450}{n} \quad p = \frac{450}{n}$

$\frac{(n-5)(p+4.5)}{n-5} = \frac{450}{n-5}$

$p + 4.5 = \frac{450}{n-5}$

$\frac{450}{n}\big(2n(n-5)\big) + \frac{9}{2}\big(2n(n-5)\big) = \frac{450}{n-5}\big(2n(n-5)\big)$

$LCD: \big(2n(n-5)\big)$

$900(n-5) + 9n(n-5) = 900n$

$900n - 4500 + 9n^2 - 45n = 900n$

$9n^2 + 855n - 4500 = 900n$

$-900n \qquad -900n$

$\frac{9n^2}{9} - \frac{45n}{9} - \frac{4500}{9} = \frac{0}{9}$

$n^2 - 5n - 500 = 0$

$(n-25)(n+20) = 0$

$n - 25 = 0 \quad n + 20 = 0$

$+25 \;\; +25 \quad -20 \;\; -20$

$n = 25 \qquad \cancel{n = -20}$

9) A factory tests the road performance of new model cars by driving them at two different rates of speed for at least 100 kilometers at each rate. The speed rates range from 50 to 70 km/hr in the lower range and from 70 to 90 km/hr in the higher range. A driver plans to test a car on an available speedway by driving it for 120 kilometers at a speed in the lower range and then driving 120 kilometers at a rate that is 20 km/hr faster. At what rates should he drive if he plans to complete the test in $3\frac{1}{2}$ hours?

r	t	d
r	t	120
$r+20$	$35-t$	120

$\frac{rt}{r}=\frac{120}{r}\quad t=\frac{120}{r}$

$\frac{(r+20)(3.5-t)}{r+20}=\frac{120}{r+20}$

$\frac{7}{2}\left(2r(r+20)\right)-\frac{120}{r}\left(2r(r+20)\right)=\frac{120}{r+20}\left(2r(r+20)\right)$

$LCD:\left(2r(r+20)\right)$

$7r(r+20)-240(r+20)=240r$

$7r^2+140r-240r-480=240r$

$7r^2-100r-480=240r$

$-240r\qquad -240r$

$7r^2-340r-4800=0$

$\frac{340\pm\sqrt{(340)^2-4(7)(-4800)}}{2(7)}=\frac{340\pm\sqrt{25000}}{14}=\frac{340\pm500}{14}=60,\cancel{-\frac{80}{7}}$

$60mph\ \&\ 80mph$

11) The rate of the current in a stream is 3 km/hr. A man rowed upstream for 3 kilometers and then returned. The round trip required 1 hour and 20 minutes. How fast was he rowing?

$1\frac{1}{3}=\frac{4}{3}$

r	t	d
r	t	3
$r-3$	$\frac{4}{3}-t$	3

$\frac{(r+3)t}{r+3}=\frac{3}{r+3}\quad t=\frac{3}{r+3}$

$\frac{(r-3)\left(\frac{4}{3}-t\right)}{r-3}=\frac{3}{r-3}$

$\frac{4}{3}-t=\frac{3}{r-3}$

$\frac{4}{3}\,3(r+3)(r-3)-\frac{3}{r+3}3(r+3)(r-3)=\frac{3}{r-3}3(r+3)(r-3)$

$LCD:3(r+3)(r-3)$

$4(r^2-9)-9(r-3)=9(r+3)$

$4r^2-36-9r+27=9r+27$

$4r^2-9r-9=9r+27$

$-9r-27\quad -9r-27$

$\frac{4r^2}{2}-\frac{18r}{2}-\frac{36}{2}=0$

$2r^2-9r-18=0$

$(2r+3)(r-6)=0$

$2r+3=0\quad r-6=0$

$-3\ \ -3\quad +6\ \ +6$

$\frac{2r}{2}=-\frac{3}{2}\qquad r=6$

$\cancel{r=-\frac{3}{2}}$

13) Two drivers are testing the same model car at speeds that differ by 20 km/hr. The one driving at the slower rate drives 70 kilometers down a speedway and returns by the same route. The one driving at the faster rate drives 76 kilometers down the speedway and returns by the same route. Both drivers leave at the same time, and the faster car returns $\frac{1}{2}$ hour earlier than the slower car. At what rates were the cars driven?

N	P	T
N	P	900
$n+3$	$p-15$	900

15) An automobile goes to a place 72 miles away and then returns, the round trip occupying 9 hours. His speed in returning is 12 miles per hour faster than his speed in going. Find the rate of speed in both going and returning.

N	P	T
N	P	900
$n+3$	$p-15$	900

17) The rate of a stream is 3 miles an hour. If a crew rows downstream for a distance of 8 miles and then back again, the round trip occupying 5 hours, what is the rate of the crew in still water?

N	P	T
N	P	900
$n+3$	$p-15$	900

19) By going 15 miles per hour faster, a train would have required 1 hour less to travel 180 miles. How fast did it travel?

N	P	T
N	P	900
$n+3$	$p-15$	900

21) If a train had traveled 5 miles an hour faster, it would have needed $1\frac{1}{2}$ hours less time to travel 150 miles. Find the rate of the train.

N	P	T
N	P	900
$n+3$	$p-15$	900