

## 9.6

1)  $x^4 - 5x^2 + 4 = 0$

$y = x^2, y^2 = x^4$

$y^2 - 5y + 4 = 0$

$(y - 4)(y - 1) = 0$

$y - 4 = 0 \quad y - 1 = 0$

$\frac{+4 \quad +4}{\quad} \quad \frac{+1 \quad +1}{\quad}$

$y = 4 \quad y = 1$

$\sqrt{x^2} = \sqrt{4} \quad x^2 = \sqrt{1}$

$x = \pm 2, \pm 1$

3)  $m^4 - 7m^2 - 8 = 0$

$y = m^2 \quad y^2 = m^4$

$y^2 - 7y - 8 = 0$

$(y - 8)(y + 1) = 0$

$y - 8 = 0 \quad y + 1 = 0$

$\frac{+8 \quad +8}{\quad} \quad \frac{-1 \quad -1}{\quad}$

$y = 8 \quad y = -1$

$\sqrt{m^2} = \sqrt{8} \quad \sqrt{m^2} = \sqrt{(-1)^2}$

$m = \pm 2\sqrt{2}, \pm i$

5)  $a^4 - 50a^2 + 49 = 0$

$y = a^2 \quad y^2 = a^4$

$y^2 - 50y + 49 = 0$

$(y - 49)(y - 1) = 0$

$y - 49 = 0 \quad y - 1 = 0$

$\frac{+49 \quad +49}{\quad} \quad \frac{+1 \quad +1}{\quad}$

$y = 49 \quad y = 1$

$\sqrt{a^2} = \sqrt{49} \quad \sqrt{a^2} = \sqrt{1}$

$a = \pm 7, \pm 1$

7)  $x^4 - 25x^2 + 144 = 0$

$y = x^2, y^2 = x^4$

$y^2 - 25y + 144 = 0$

$(y - 9)(y - 16) = 0$

$y - 9 = 0 \quad y - 16 = 0$

$\frac{+9 \quad +9}{\quad} \quad \frac{+16 \quad +16}{\quad}$

$y = 9 \quad y = 16$

$\sqrt{x^2} = \sqrt{9} \quad \sqrt{x^2} = \sqrt{16}$

$x = \pm 3, \pm 4$

$$\begin{aligned}
9) \quad & m^4 - 20m^2 + 64 = 0 \\
& y = m^2 \quad y^2 = m^4 \\
& y^2 - 20y + 64 = 0 \\
& (y - 4)(y - 16) = 0 \\
& y - 4 = 0 \quad y - 16 = 0 \\
& \frac{+4 \quad +4}{y = 4} \quad \frac{+16 \quad +16}{y = 16} \\
& \sqrt{m^2} = \sqrt{4} \quad \sqrt{m^2} = \sqrt{16} \\
& m = \pm 2, \pm 4
\end{aligned}$$

$$\begin{aligned}
11) \quad & z^6 - 216 = 19z^3 \\
& y = z^3 \quad y^2 = z^6 \\
& y^2 - 216 = 19y \\
& \frac{-19y \quad -19y}{y^2 - 19y - 216 = 0} \\
& (y - 27)(y + 8) = 0 \\
& y - 27 = 0 \quad y + 8 = 0 \\
& \frac{+27 \quad +27}{y = 27} \quad \frac{-8 \quad -8}{y = -8} \\
& z^3 = 27 \quad z^3 = -8 \\
& \frac{-27 \quad -27}{z^3 - 27 = 0} \quad \frac{+8 \quad +8}{z^3 - 8 = 0} \\
& (z - 3)(z^2 + 3z + 9) = 0 \\
& z - 3 = 0 \quad z^2 + 3z + 9 = 0 \\
& \frac{+3 \quad +3}{z = 3} \quad \frac{-3 \pm \sqrt{3^2 - 4(1)(9)}}{2(1)} = \frac{2 \pm \sqrt{-27}}{2} = \frac{-3 \pm 3i\sqrt{3}}{2} \\
& (z + 2)(z^2 - 2z + 4) = 0 \\
& z + 2 = 0 \quad z^2 - 2z + 4 = 0 \\
& \frac{-2 \quad -2}{z = -2} \quad \frac{2 \pm \sqrt{(-2)^2 - 4(1)(4)}}{2} = \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2i\sqrt{3}}{2} = 1 \pm i\sqrt{3} \\
& z = 3, \frac{-3 \pm 3i\sqrt{3}}{2}, -2, 1 \pm i\sqrt{3}
\end{aligned}$$

$$13) 6z^4 - z^2 = 12$$

$$y = z^2, y^2 = z^4$$

$$6y^2 - y = 12$$

$$\begin{array}{r} -12 \quad -12 \\ \hline 6y^2 - y - 12 = 0 \end{array}$$

$$6y^2 - y - 12 = 0$$

$$(3y + 4)(2y - 3) = 0$$

$$3y + 4 = 0 \quad 2y - 3 = 0$$

$$\begin{array}{r} -4 \quad -4 \quad +3 \quad +3 \\ \hline \frac{3y}{3} = -\frac{4}{3} \quad \frac{2y}{2} = \frac{3}{2} \end{array}$$

$$\frac{3y}{3} = -\frac{4}{3} \quad \frac{2y}{2} = \frac{3}{2}$$

$$y = -\frac{4}{3}, \frac{3}{2}$$

$$\sqrt{z^2} = \sqrt{-\frac{4}{3} \left(\frac{\sqrt{3}}{\sqrt{3}}\right)} \quad \sqrt{z^2} = \sqrt{\frac{3}{2} \left(\frac{\sqrt{2}}{\sqrt{2}}\right)}$$

$$z = \frac{\pm 2i\sqrt{3}}{3}, \frac{\pm\sqrt{6}}{2}$$

$$15) x^{\frac{2}{3}} - 35 = 2x^{\frac{1}{3}}$$

$$y = x^{\frac{1}{3}}, y^2 = x^{\frac{2}{3}}$$

$$y^2 - 35 = 2y$$

$$\begin{array}{r} -2y \quad -2y \\ \hline y^2 - 2y - 35 = 0 \end{array}$$

$$y^2 - 2y - 35 = 0$$

$$(y - 7)(y + 5) = 0$$

$$y - 7 = 0 \quad y + 5 = 0$$

$$\begin{array}{r} +7 \quad +7 \quad -5 \quad -5 \\ \hline y = 7, \quad y = -5 \end{array}$$

$$y = 7, \quad y = -5$$

$$x^{\frac{1}{3}} = -5 \quad x^{\frac{1}{3}} = 7$$

$$\left(\sqrt[3]{x}\right)^3 = (-5)^3 \left(\sqrt[3]{x}\right)^3 = 7^3$$

$$x = -125, 343$$

$$17) y^{-6} + 7y^{-3} = 8$$

$$z = y^{-3} \quad z^2 = y^{-6}$$

$$z^2 + 7z = 8$$

$$\begin{array}{r} -8 \quad -8 \\ \hline z^2 + 7z - 8 = 0 \end{array}$$

$$z^2 + 7z - 8 = 0$$

$$(z + 8)(z - 1) = 0$$

$$z + 8 = 0 \quad z - 1 = 0$$

$$\begin{array}{r} -8 \quad -8 \quad +1 \quad +1 \\ \hline z = -8, \quad z = 1 \end{array}$$

$$z = -8, \quad z = 1$$

$$y^{-3} = -8, y^{-3} = 1$$

$$(y^3) \left(\frac{1}{y^3}\right) = -8(y^3) \quad (y^3) \frac{1}{y^3} = 1(y^3)$$

$$1 = -8y^3 \quad 1 = y^3$$

$$\frac{+8y^3}{+8y^3} + \frac{+8y^3}{+8y^3} \quad \frac{-1}{-1} \frac{-1}{-1}$$

$$8y^3 + 1 = 0 \quad 0 = y^3 - 1$$

$$(2y + 1)(4y^2 - 2y + 1) = 0 \quad 0 = (y - 1)(y^2 + y + 1)$$

$$2y + 1 = 0 \quad 4y^2 - 2y + 1 = 0 \quad y - 1 = 0 \quad y^2 + y + 1 = 0$$

$$\frac{-1}{2} \frac{-1}{2} \frac{2 \pm \sqrt{(-2)^2 - 4(4)(1)}}{2(4)} \quad \frac{+1}{2(1)} \frac{+1}{2(1)} \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$

$$\frac{2y}{2} = \frac{-1}{2} \quad \frac{2 \pm \sqrt{-12}}{8} \quad y = 1 \quad \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

$$y = -\frac{1}{2} \quad \frac{2 \pm 2i\sqrt{3}}{8} = \frac{1 \pm i\sqrt{3}}{4}$$

$$y = -\frac{1}{2}, \frac{1 \pm i\sqrt{3}}{4}, 1, \frac{-1 \pm i\sqrt{3}}{2}$$

19)  $x^4 - 2x^2 - 3 = 0$

$$y = x^2 \quad y^2 = x^4$$

$$y^2 - 2y - 3 = 0$$

$$(y - 3)(y + 1) = 0$$

$$y - 3 = 0 \quad y + 1 = 0$$

$$\frac{+3}{y} \frac{+3}{y} \quad \frac{-1}{y} \frac{-1}{y}$$

$$\sqrt{x^2} = \sqrt{3} \quad \sqrt{x^2} = \sqrt{-1}$$

$$x = \pm\sqrt{3}, \pm i$$

21)  $2x^4 - 5x^2 + 2 = 0$

$$y = x^2, y^2 = x^4$$

$$2y^2 - 5y + 2 = 0$$

$$(2y - 1)(y - 2) = 0$$

$$2y - 1 = 0 \quad y - 2 = 0$$

$$\frac{+1}{2} \frac{+1}{2} \quad \frac{+2}{y} \frac{+2}{y}$$

$$\frac{2y}{2} = \frac{1}{2} \quad y = 2$$

$$y = \frac{1}{2}$$

$$\sqrt{x^2} = \sqrt{\frac{1}{2}} \left(\frac{\sqrt{2}}{\sqrt{2}}\right) \quad \sqrt{x^2} = \sqrt{2}$$

$$x = \pm \frac{\sqrt{2}}{2}, \pm\sqrt{2}$$

$$23) x^4 - 9x^2 + 8 = 0$$

$$y = x^2, y^2 = x^4$$

$$y^2 - 9y + 8 = 0$$

$$(y - 8)(y - 1) = 0$$

$$y - 8 = 0 \quad y - 1 = 0$$

$$\frac{+8 + 8}{y = 8} \quad \frac{+1 + 1}{y = 1}$$

$$\sqrt{x^2} = \sqrt{8} \quad \sqrt{x^2} = \sqrt{1}$$

$$x = \pm 2\sqrt{2}, \pm 1$$

$$25) 8x^6 - x^3 + 1 = 0$$

$$y = x^3, y^2 = x^6$$

$$8y^2 - y + 1 = 0$$

$$(8y - 1)(y - 1) = 0$$

$$8y - 1 = 0 \quad y - 1 = 0$$

$$\frac{+1 + 1}{\frac{8y}{8} = \frac{1}{8}} \quad \frac{+1 + 1}{y = 1}$$

$$y = \frac{1}{8}$$

$$(8)x^3 = \frac{1}{8}(8)$$

$$8x^3 = 1$$

$$\frac{-1 - 1}{8x^3 - 1 = 0}$$

$$(2x - 1)(4x^2 + 2x + 1) = 0$$

$$2x - 1 = 0 \quad 4x^2 + 2x + 1 = 0$$

$$\frac{+1 + 1}{\frac{2x}{2} = \frac{1}{2}} \quad \frac{-2 \pm \sqrt{2^2 - 4(4)(1)}}{2(4)}$$

$$\frac{2x}{2} = \frac{1}{2} \quad \frac{-2 \pm \sqrt{-12}}{8}$$

$$x = \frac{1}{2} \quad \frac{-2 \pm 2i\sqrt{3}}{8} = \frac{-1 \pm i\sqrt{3}}{4}$$

$$x = \frac{1}{2}, \frac{-1 \pm i\sqrt{3}}{4}, 1, \frac{-1 \pm i\sqrt{3}}{2}$$

$$x^3 = 1$$

$$\frac{-1 - 1}{x^3 - 1 = 0}$$

$$(x - 1)(x^2 + x + 1) = 0$$

$$x - 1 = 0 \quad x^2 + x + 1 = 0$$

$$\frac{+1 + 1}{\frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}}$$

$$x = 1 \quad \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

$$27) x^8 - 17x^4 + 16 = 0$$

$$y = x^4, y^2 = x^8$$

$$y^2 - 17y + 16 = 0$$

$$(y - 16)(y - 1) = 0$$

$$y - 16 = 0 \quad y - 1 = 0$$

$$\frac{+16 + 16}{y = 16} \quad \frac{+1 + 1}{y = 1}$$

$$\begin{aligned}
x^4 &= 16 \\
-16 & \quad -16 \\
x^4 - 16 &= 0 \\
(x^2 + 4)(x^2 - 4) &= 0 \\
x^2 + 4 = 0 & \quad x^2 - 4 = 0 \\
\frac{-4}{x^2} & \quad \frac{-4}{x^2} \quad \frac{+4}{x^2} \quad \frac{+4}{x^2} \\
\sqrt{x^2} &= \sqrt{-4} \quad \sqrt{x^2} = \sqrt{4} \\
x &= \pm 2i \quad x = \pm 2 \\
x &= \pm 2i, \pm 2, \pm i, \pm 1
\end{aligned}$$

$$\begin{aligned}
x^4 &= 1 \\
-1 & \quad -1 \\
x^4 - 1 &= 0 \\
(x^2 - 1)(x^2 + 1) &= 0 \\
x^2 - 1 = 0 & \quad x^2 + 1 = 0 \\
\frac{+1}{x^2} & \quad \frac{+1}{x^2} \quad \frac{-1}{x^2} \quad \frac{-1}{x^2} \\
\sqrt{x^2} &= \sqrt{1} \quad \sqrt{x^2} = \sqrt{-1} \\
x &= \pm 1 \quad x = \pm i
\end{aligned}$$

29)  $(y + b)^2 - 4(y + b) = 21$   
 $z = (y + b), z^2 = (y + b)^2$   
 $z^2 - 4z = 21$   
 $\frac{-21}{z^2} \quad \frac{-21}{z^2}$   
 $z^2 - 4z - 21 = 0$   
 $(z - 7)(z + 3) = 0$   
 $z - 7 = 0 \quad z + 3 = 0$   
 $\frac{+7}{z} \quad \frac{+7}{z} \quad \frac{-3}{z} \quad \frac{-3}{z}$   
 $z = 7 \quad z = -3$   
 $y + b = 7 \quad y + b = -3$   
 $\frac{-b}{y} \quad \frac{-b}{y} \quad \frac{-b}{y} \quad \frac{-b}{y}$   
 $y = 7 - b \quad y = -3 - b$

31)  $(y + 2)^2 - 6(y + 2) = 16$   
 $z = y + 2, z^2 = (y + 2)^2$   
 $z^2 - 6z = 16$   
 $\frac{-16}{z^2} \quad \frac{-16}{z^2}$   
 $z^2 - 6z - 16 = 0$   
 $(z - 8)(z + 2) = 0$   
 $z - 8 = 0 \quad z + 2 = 0$   
 $\frac{+8}{z} \quad \frac{+8}{z} \quad \frac{-2}{z} \quad \frac{-2}{z}$   
 $z = 8 \quad z = -2$   
 $y + 2 = 8 \quad y + 2 = -2$   
 $\frac{-2}{y} \quad \frac{-2}{y} \quad \frac{-2}{y} \quad \frac{-2}{y}$   
 $y = 6 \quad y = -4$

$$33) (x - 3)^2 - 2(x - 3) = 35$$

$$y = (x - 3), y^2 = (x - 3)^2$$

$$y^2 - 2y = 35$$

$$\underline{-35 \quad -35}$$

$$y^2 - 2y - 35 = 0$$

$$(y - 7)(y + 5) = 0$$

$$y - 7 = 0 \quad y + 5 = 0$$

$$\underline{+7 \quad +7} \quad \underline{-5 \quad -5}$$

$$y = 7 \quad y = -5$$

$$x - 3 = 7 \quad x - 3 = -5$$

$$\underline{+3 \quad +3} \quad \underline{+3 \quad +3}$$

$$x = 10, -2$$

$$35) (r - 1)^2 - 8(r - 1) = 20$$

$$y = (r - 1), y^2 = (r - 1)^2$$

$$y^2 - 8y = 20$$

$$\underline{-20 \quad -20}$$

$$y^2 - 8y - 20 = 0$$

$$(y - 10)(y + 2) = 0$$

$$y - 10 = 0 \quad y + 2 = 0$$

$$\underline{+10 \quad +10} \quad \underline{-2 \quad -2}$$

$$y = 10 \quad y = -2$$

$$r - 1 = 10 \quad r - 1 = -2$$

$$\underline{+1 \quad +1} \quad \underline{+1 \quad +1}$$

$$r = 11, -1$$

$$37) 3(y + 1)^2 - 14(y + 1) = 5$$

$$z = (y + 1), z^2 = (y + 1)^2$$

$$3z^2 - 14z = 5$$

$$\underline{-5 \quad -5}$$

$$3z^2 - 14z - 5 = 0$$

$$(3z + 1)(z - 5) = 0$$

$$3z + 1 = 0 \quad z - 5 = 0$$

$$\underline{-1 \quad -1} \quad \underline{+5 \quad +5}$$

$$\frac{3z}{3} = \frac{-1}{3} \quad z = 5$$

$$z = -\frac{1}{3}$$

$$y + 1 = -\frac{1}{3} \quad y + 1 = 5$$

$$\underline{-1 \quad -1} \quad \underline{-1 \quad -1}$$

$$y = -\frac{4}{3}, 4$$

$$39) (3x^2 - 2x)^2 + 5 = 6(3x^2 - 2x)$$

$$y = (3x^2 - 2x), y^2 = (3x^2 - 2x)^2$$

$$y^2 + 5 = 6y$$

$$\underline{-6y \quad -6y}$$

$$y^2 - 6y + 5 = 0$$

$$(y - 1)(y - 5) = 0$$

$$y - 1 = 0 \quad y - 5 = 0$$

$$\underline{+1 \quad +1 \quad +5 \quad +5}$$

$$y = 1 \quad y = 5$$

$$3x^2 - 2x = 1$$

$$3x^2 - 2x = 5$$

$$\underline{-1 \quad -1}$$

$$\underline{-5 \quad -5}$$

$$3x^2 - 2x - 1 = 0$$

$$3x^2 - 2x - 5 = 0$$

$$(3x - 5)(x + 1) = 0 \quad (3x + 1)(x - 1) = 0$$

$$3x - 5 = 0 \quad x + 1 = 0 \quad 3x + 1 = 0 \quad x - 1 = 0$$

$$\underline{+5 \quad +5 \quad -1 \quad -1 \quad -1 \quad -1 \quad +1 \quad +1}$$

$$\frac{3x}{3} = \frac{5}{3} \quad x = -1 \quad \frac{3x}{3} = \frac{-1}{3} \quad x = 1$$

$$x = \frac{5}{3}, -1, -\frac{1}{3}, 1$$

$$41) 2(3x + 1)^{\frac{2}{3}} - 5(3x + 1)^{\frac{1}{3}} = 88$$

$$y = (3x + 1)^{\frac{1}{3}}, y^2 = (3x + 1)^{\frac{2}{3}}$$

$$2y^2 - 5y = 88$$

$$\underline{-88 \quad -88}$$

$$2y^2 - 5y - 88 = 0$$

$$(2y + 11)(y - 8) = 0$$

$$2y + 11 = 0 \quad y - 8 = 0$$

$$\underline{-11 \quad -11 \quad +8 \quad +8}$$

$$\frac{2y}{2} = \frac{-11}{2} \quad y = 8$$

$$y = -\frac{11}{2}$$

$$(3x + 1)^{\frac{1}{3}} = -\frac{11}{2}$$

$$(3x + 1)^{\frac{1}{3}} = 8$$

$$(\sqrt[3]{3x + 1})^3 = \left(-\frac{11}{2}\right)^3$$

$$(\sqrt[3]{3x + 1})^3 = 8^3$$

$$3x + 1 = -\frac{1331}{8}$$

$$3x + 1 = 512$$

$$\underline{-1 \quad -1}$$

$$\underline{-1 \quad -1}$$

$$\frac{3x}{3} = \left(-\frac{1329}{8}\right)$$

$$\frac{3x}{3} = \frac{511}{3}$$

$$x = \frac{1329}{24}, \frac{511}{3}$$



$$43) (x^2 + 2x)^2 - 2(x^2 + 2x) = 3$$

$$y = (x^2 + 2x), y^2 = (x^2 + 2x)^2$$

$$y^2 - 2y = 3$$

$$\underline{-3 \quad -3}$$

$$y^2 - 2y - 3 = 0$$

$$(y - 3)(y + 1) = 0$$

$$y - 3 = 0 \quad y + 1 = 0$$

$$\underline{+3 \quad +3} \quad \underline{-1 \quad -1}$$

$$y = 3 \quad y = -1$$

$$x^2 + 2x = 3 \quad x^2 + 2x = -1$$

$$\underline{-3 \quad -3} \quad \underline{+1 \quad +1}$$

$$x^2 + 2x - 3 = 0 \quad x^2 + 2x + 1 = 0$$

$$(x + 3)(x - 1) = 0 \quad (\sqrt{(x + 1)^2} = \sqrt{0})$$

$$x + 3 = 0 \quad x - 1 = 0 \quad x + 1 = 0$$

$$\underline{-3 \quad -3} \quad \underline{+1 \quad +1} \quad \underline{-1 \quad -1}$$

$$x = -3, 1, -1$$

$$45) (2x^2 - x)^2 - 4(2x^2 - x) + 3 = 0$$

$$y = (2x^2 - x), y^2 = (2x^2 - x)^2$$

$$y^2 - 4y + 3 = 0$$

$$(y - 3)(y - 1) = 0$$

$$y - 3 = 0 \quad y - 1 = 0$$

$$\underline{+3 \quad +3} \quad \underline{+1 \quad +1}$$

$$y = 3 \quad y = 1$$

$$2x^2 - x = 3$$

$$2x^2 - x = 1$$

$$\underline{-3 \quad -3}$$

$$\underline{-1 \quad -1}$$

$$2x^2 - x - 3 = 0$$

$$2x^2 - x - 1 = 0$$

$$(2x - 3)(x + 1) = 0$$

$$(2x + 1)(x - 1) = 0$$

$$2x - 3 = 0 \quad x + 1 = 0$$

$$2x + 1 = 0 \quad x - 1 = 0$$

$$\underline{+3 \quad +3} \quad \underline{-1 \quad -1}$$

$$\underline{-1 \quad -1} \quad \underline{+1 \quad +1}$$

$$\frac{2x}{2} = \frac{3}{2} \quad x = -1$$

$$\frac{2x}{2} = \frac{-1}{2} \quad x = 1$$

$$x = \frac{3}{2}, -1, -\frac{1}{2}, 1$$