

- 1) Bills father can paint a room in two hours less than Bill can paint it. Working together they can complete the job in two hours and 24 minutes. How much time would each require working alone?

$$\text{Father: } x - 2$$

$$\text{Bill: } x$$

$$\text{Team: } 2\frac{24}{60} = 2\frac{2}{5} = \frac{12}{5}$$

$$\frac{1}{x-2} 12x(x-2) + \frac{1}{x} 12x(x-2) = \frac{5}{12} 12x(x-2)$$

$$\text{LCD: } 12x(x-2)$$

$$12x + 12(x-2) = 5x(x-2)$$

$$12x + 12x - 24 = 5x^2 - 10x$$

$$24x - 24 = 5x^2 - 10$$

$$\frac{-24x + 24 \quad -24x + 24}{\phantom{0 = 5x^2 - 34x + 24}}$$

$$0 = 5x^2 - 34x + 24$$

$$0 = (5x - 4)(x - 6)$$

$$5x - 4 = 0 \quad x - 6 = 0$$

$$\frac{+4 \quad +4}{\phantom{\frac{5x}{5} = \frac{4}{5}}} \quad \frac{+6 \quad +6}{\phantom{x = 6}}$$

$$\frac{5x}{5} = \frac{4}{5} \quad x = 6$$

$$\cancel{x = 4}$$

$$\boxed{\text{Bill: 6hr, Father: 4}}$$

- 3) Jack can wash and wax the family car in one hour less than Bob can. The two working together can complete the job in  $1\frac{1}{5}$  hours. How much time would each require if they worked alone?

$$\text{Jack: } x - 1$$

$$\text{Bob: } x$$

$$\text{Team: } 1\frac{1}{5} = \frac{6}{5}$$

$$\frac{1}{x-1} (6x(x-1)) + \frac{1}{x} (6x(x-1)) = \frac{6}{5} (6x(x-1))$$

$$\text{LCD: } 6x(x-1)$$

$$6x + 6(x-1) = 5x(x-1)$$

$$6x + 6x - 6 = 5x^2 - 5x$$

$$12x - 6 = 5x^2 - 5x$$

$$\frac{-12x + 6 \quad -12x + 6}{\phantom{0 = 5x^2 - 17x + 6}}$$

$$0 = 5x^2 - 17x + 6$$

$$0 = (5x - 2)(x - 3)$$

$$5x - 2 = 0 \quad x - 3 = 0$$

$$\frac{+2 \quad +2}{\phantom{\frac{5x}{5} = \frac{2}{5}}} \quad \frac{+3 \quad +3}{\phantom{x = 3}}$$

$$\frac{5x}{5} = \frac{2}{5} \quad x = 3$$

$$\cancel{x = 4}$$

$$\boxed{\text{Bob: 3hr, Jack: 2hr}}$$

- 5) Working alone it takes John 8 hours longer than Carlos to do a job. Working together they can do the job in 3 hours. How long will it take each to do the job working alone

$$\text{John: } x + 8 \qquad \frac{1}{x+8} 3x(x+8) + \frac{1}{x} 3x(x+8) = \frac{1}{3} 3x(x+8)$$

$$\text{Carlos: } x \qquad \text{LCD: } 3x(x+8)$$

$$\text{Team: } 3 \qquad 3x + 3(x+8) = x(x+8)$$

$$3x + 3x + 24 = x^2 + 8x$$

$$6x + 24 = x^2 + 8x$$

$$\frac{-6x - 24}{-6x - 24} \qquad \frac{-6x - 24}{-6x - 24}$$

$$0 = x^2 + 2x - 24$$

$$0 = (x+6)(x-4)$$

$$x+6 = 0 \quad x-4 = 0$$

$$\frac{-6 \quad -6}{x} \qquad \frac{+4 \quad +4}{x} \qquad x = 4$$

$$\cancel{x} > \cancel{-6} \qquad x = 4$$

**Carlos: 4hr, John: 12hr**

- 7) A can do a piece of work in 4 days and B can do it in half the time. How long will it take them to do the work together?

$$A: 4 \qquad \frac{1}{4}(4x) + \frac{1}{2}(4x) = \frac{1}{x}(4x)$$

$$B: 2 \qquad \text{LCD: } 4x$$

$$\text{Team: } x \qquad x + 2x = 4$$

$$\frac{3x}{3} = \frac{4}{3}$$

$$x = .133$$

**(1 hr., 20 min.)**

- 9) If A can do a piece of work in 24 days and A and B together can do it in 6 days, how long would it take B to do the work alone?

$$A: 24 \qquad \frac{1}{24}(24x) + \frac{1}{x}(24x) = \frac{1}{6}(24x)$$

$$B: x \qquad \text{LCD: } 24x$$

$$\text{Team: } 6 \qquad x + 24 = 4x$$

$$\frac{-x}{-x} \qquad \frac{-x}{-x}$$

$$\frac{24}{3} = \frac{3x}{3}$$

$$8 = x$$

**8 days**

- 11) If Sam can do a certain job in 3 days, while it takes Fred 6 days to do the same job, how long will it take them, working together, to complete the job?

$$\begin{array}{l}
 \text{Sam: } 3 \\
 \text{Fred: } 6 \\
 \text{Team: } x
 \end{array}
 \qquad
 \begin{array}{l}
 \frac{1}{3}(6x) + \frac{1}{6}(6x) = \frac{1}{x}(6x) \\
 \text{LCD: } 6x \\
 2x + x = 6 \\
 \frac{3x}{3} = \frac{6}{3} \\
 x = \boxed{2 \text{ days}}
 \end{array}$$

- 13) Two people working together can complete a job in 6 hours. If one of them works twice as fast as the other, how long would it take the faster person, working alone, to do the job?

$$\begin{array}{l}
 A = x \\
 B = 2x \\
 \text{Team: } 6
 \end{array}
 \qquad
 \begin{array}{l}
 \frac{1}{x}(6x) + \frac{1}{2x}(6x) = \frac{1}{6}(6x) \\
 6 + 3 = x \\
 9 = x \\
 \boxed{A = 9hr}, B = 18hr
 \end{array}$$

- 15) A water tank can be filled by an inlet pipe in 8 hours. It takes twice that long for the outlet pipe to empty the tank. How long will it take to fill the tank if both pipes are open?

$$\begin{array}{l}
 \text{In: } 8 \\
 \text{Out: } -16 \\
 \text{Team: } x
 \end{array}
 \qquad
 \begin{array}{l}
 \frac{1}{8}(16x) - \frac{1}{16}(16x) = \frac{1}{x}(16x) \\
 2x - x = 16 \\
 x = \boxed{16hr}
 \end{array}$$

- 17) It takes 10 hours to fill a pool with the inlet pipe. It can be emptied in 15 hrs. with the outlet pipe. If the pool is half full to begin with, how long will it take to fill it from there if both pipes are open?

$$\begin{array}{l}
 \text{In: } 10 \\
 \text{Out: } -15 \\
 \text{Team: } x
 \end{array}
 \qquad
 \begin{array}{l}
 \frac{1}{10}(30x) - \frac{1}{15}(30x) = \frac{1}{x}(30x) \\
 3x - 2x = 30 \\
 x = 30 \\
 \frac{1}{2}(30) = \boxed{15 \text{ hr}}
 \end{array}$$

- 19) A sink has two faucets, one for hot water and one for cold water. The sink can be filled by a cold-water faucet in 3.5 minutes. If both faucets are open, the sink is filled in 2.1 minutes. How long does it take to fill the sink with just the hot-water faucet open?

$$\begin{array}{l}
 \text{Hot: } x \\
 \text{Cold: } 3.5 = \frac{35}{10} = \frac{7}{2} \\
 \text{Team: } 2.1 = \frac{21}{10}
 \end{array}
 \qquad
 \begin{array}{l}
 \frac{1}{x}(21x) + \frac{2}{7}(21x) = \frac{10}{21}(21x) \\
 \text{LCD: } 21x \\
 21 + 6x = 10x \\
 -6x \quad -6x \\
 \frac{21}{4} = \frac{4x}{4} \\
 \boxed{5.25 \text{ hr}} = x
 \end{array}$$

- 21) A tank can be emptied by any one of three caps. The first can empty the tank in 20 minutes while the second takes 32 minutes. If all three working together could empty the tank in  $8\frac{8}{59}$  minutes, how long would the third take to empty the tank?

$$\text{First: } 20 \qquad \frac{1}{20}(480x) + \frac{1}{32}(480x) + \frac{1}{x}(480x) = \frac{59}{480}(480x)$$

$$\text{Second: } 32 \qquad \text{LCD: } 480x$$

$$\text{Third: } x \qquad 24x + 15x + 480 = 59x$$

$$\text{Team: } 8\frac{8}{59} = \frac{480}{59} \qquad 39x + 480 = 59x$$

$$\frac{-39x}{-39x} \qquad \frac{-39x}{-39x}$$

$$\frac{480}{20} = \frac{20x}{20}$$

$$24 = x$$

$$\boxed{24 \text{ min.}}$$

- 23) Sam takes 6 hours longer than Susan to wax a floor. Working together they can wax the floor in 4 hours. How long will it take each of them working alone to wax the floor?

$$\text{Sam: } x + 6 \qquad \frac{1}{x+6}(4x(x+6)) + \frac{1}{x}(4x(x+6)) = \frac{1}{4}(4x(x+6))$$

$$\text{Susan: } x \qquad \text{LCD: } (4x(x+6))$$

$$\text{Team: } 4 \qquad 4x + 4(x+6) = x(x+6)$$

$$4x + 4x + 24 = x^2 + 6x$$

$$8x + 24 = x^2 + 6x$$

$$\frac{-8x - 24}{-8x - 24} \qquad \frac{-8x - 24}{-8x - 24}$$

$$0 = x^2 - 2x - 24$$

$$0 = (x-6)(x+4)$$

$$x - 6 = 0 \quad x + 4 = 0$$

$$\frac{+6 \quad +6}{x-6} \qquad \frac{-4 \quad -4}{x-4}$$

$$\cancel{x-6} \qquad x = -4$$

$$\boxed{\text{Susan} = 6\text{hr}, \text{Sam} = 12\text{hr}}$$

- 25) It takes Sally  $10\frac{1}{2}$  minutes longer than Patricia to clean up their dorm room. If they work together they can clean it in 5 minutes. How long will it take each of them if they work alone?

$$\text{Sally: } x + 10.5 = x + \frac{21}{2} = \frac{2x+21}{2} \quad \frac{2}{2x+21} (5x(2x+21)) + \frac{1}{x} (5x(2x+21)) = \frac{1}{5} (5x(2x+21))$$

$$\text{Patricia: } x$$

$$\text{Team: } 5$$

$$\text{LCD: } (5x(2x+21))$$

$$10x + 5(2x+21) = x(2x+21)$$

$$10x + 10x + 105 = 2x^2 + 21x$$

$$20x + 105 = 2x^2 + 21x$$

$$\begin{array}{r} -20x - 105 \qquad -20x - 105 \\ \hline \end{array}$$

$$0 = 2x^2 + x - 105$$

$$0 = (2x+15)(x-7)$$

$$2x+15=0 \quad x-7=0$$

$$\begin{array}{r} -15 \quad -15 \quad +7 \quad +7 \\ \hline \end{array}$$

$$\frac{2x}{2} = -\frac{15}{2} \qquad x = 7$$

$$x = \cancel{-\frac{15}{2}}$$

$$\boxed{\text{Patricia} = 7\text{min.}, \text{Sally} = 17.5\text{min.}}$$

- 27) Secretary A takes 6 minutes longer than Secretary B to type 10 pages of manuscript. If they divide the job and work together it will take them  $8\frac{3}{4}$  minutes to type 10 pages. How long will it take each working alone to type the 10 pages?

$$A: x + 6$$

$$B: x$$

$$\text{Team: } 8\frac{3}{4} = \frac{35}{4}$$

$$\frac{1}{x+6} (35x(x+6)) + \frac{1}{x} (35x(x+6)) = \frac{4}{35} (35x(x+6))$$

$$\text{LCD: } (35x(x+6))$$

$$35x + 35(x+6) = 4x(x+6)$$

$$35x + 35x + 210 = 4x^2 + 24x$$

$$70x + 210 = 4x^2 + 24x$$

$$\begin{array}{r} -70x - 210 \qquad -70x - 210 \\ \hline \end{array}$$

$$\frac{0}{2} = \frac{4x^2}{2} - \frac{46x}{2} - \frac{210}{2}$$

$$0 = 2x^2 - 23x - 105$$

$$0 = (2x+7)(x-15)$$

$$2x+7=0 \quad x-15=0$$

$$\begin{array}{r} -7 \quad -7 \quad +15 \quad +15 \\ \hline \end{array}$$

$$\frac{2x}{2} = -\frac{7}{2} \qquad x = 15$$

$$x = \cancel{-\frac{7}{2}}$$

$$\boxed{B: 15\text{hr}, A: 21\text{hr}}$$